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Managing Cognitive Load through Differentiated Instructional Processes to Enhance Mathematical Problem-Solving Skills

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A R T I C L E I N F O

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A B S T R A C T

Differentiated instruction (DI) has been identified as a strategy to accommodate the diversity of student characteristics; however, empirical evidence regarding its effect on mathematical problem-solving ability and cognitive load, particularly within the framework of Cognitive Load Theory (CLT), remains limited and inconclusive. This study examined the effect of CLT-based differentiated instruction in which worked-examples, faded-examples, and problem-examples were assigned according to students' prior knowledge level on mathematical problem-solving ability and cognitive load. A randomized pretest-posttest control group design was employed involving 84 students (aged 14–15 years) in Serang, Indonesia, who were randomly assigned to either the DI group (n = 42) or the non-DI group (n = 42). Results indicated that the DI group demonstrated significantly greater improvement in mathematical problem-solving ability and a significantly greater reduction in cognitive load compared to the non-DI group. These findings provide empirical support for the effectiveness of CLT-based differentiated instruction in enhancing mathematical problem-solving ability while simultaneously reducing students' cognitive load, with implications for the design of adaptive instructional interventions in heterogeneous classrooms.

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INTRODUCTION

Mathematics is a fundamental discipline for understanding various domains, such as science, engineering, technology, and economics (Nakakoji & Wilson, 2020). One of the core objectives of mathematics education is the development of problem-solving skills (English & Gainsburg, 2016; Lester, 2013; NCTM, 2000, 2014). This ability constitutes an essential 21st-century skill that enables the application of mathematics in real-life contexts (Amalina et al., 2026) and is categorized as a higher-order thinking skill (Alashwal & Barham, 2025; Brookhart & Nitko, 2011; Greiff et al., 2021) because it involves complex cognitive processes, such as planning, evaluation, and the flexible application of strategies to produce appropriate solutions (Alashwal & Barham, 2025).

Problem-solving performance is highly dependent on prior knowledge, both conceptual and procedural (Booth & Davenport, 2013; H Süß & Kretzschmar, 2018), as well as on the structure of knowledge and the depth of the problem solver's experience (López et al., 2007). Without sufficient prior knowledge, individuals tend to employ random search strategies and evaluate the effectiveness of each step through trial and error (Sweller, 2024), making them more susceptible to difficulties due to the limited capacity of working memory in processing new information (Chen et al., 2015; Aini & Retnowati, 2023). In contrast, individuals with well-developed prior knowledge are more efficient in understanding and solving problems because they have constructed well-organized schemas in long-term memory (Sweller, 2024; Sweller et al., 2011). The crucial role of prior knowledge in determining problem-solving success is systematically explained by Cognitive Load Theory (CLT).

Cognitive Load Theory, which is grounded in our understanding of human cognitive architecture, emphasizes the importance of designing instruction that takes into account the limited capacity of working memory, so that the information processed does not impose a cognitive load exceeding this capacity, which could hinder learning processes (Santosa et al., 2018; Sweller et al., 1998, 2011). The theoretical implications of CLT have led to the emergence of various cognitive load effects in instructional design, such as the worked-example effect and the expertise reversal effect. The worked-example effect refers to the finding that for novice learners, learning through worked examples is more effective than solving problems independently, as it reduces extraneous cognitive load and allows learners to focus their cognitive resources on understanding the structure of solutions (Sweller et al., 1998, 2011; Santosa & Filiz, 2025). However, the effectiveness of worked examples is conditional; for learners with higher levels of expertise, studying worked examples is no longer effective and may even be less beneficial than problem-solving approaches (Kalyuga et al., 2001). This occurs because fully presented examples can introduce informational redundancy, thereby increasing cognitive load, a phenomenon known as the expertise reversal effect.

In line with these findings, several studies have focused on identifying appropriate methods to facilitate learners who are transitioning from novice to higher levels of expertise in achieving independent problem-solving. In this regard, the faded-example approach has been introduced and has been empirically proven to be effective Atkinson et al., 2003; Renkl & Atkinson, 2000; Renkl et al., 2002, 2004). The fading method involves initially providing learners with fully worked examples, after which the solution steps are gradually removed one by one until learners are able to engage in independent problem-solving (ter Vrugte et al., 2017).

The pedagogical implications of differences in the effectiveness of instructional design based on learners' levels of expertise highlight the urgency of implementing instruction that provides optimal learning opportunities according to students' abilities, given that students within a single classroom possess diverse levels of competence (Jager et al., 2022). Therefore, an appropriate instructional strategy is for teachers to implement Differentiated Instruction (DI), an approach in which teachers proactively plan and implement instructional adaptations to address the diverse characteristics of learners in the classroom (Smale-Jacobse et al., 2019; Suprayogi et al., 2017; Tomlinson et al., 2003; Whitley et al., 2019). For example, teachers may adjust learning objectives, activities, content, and levels of task difficulty (Jager et al., 2022; Tomlinson et al., 2003).

A recent study conducted by Aini and Retnowati (2023) designed differentiated instruction based on Cognitive Load Theory by combining three methods tailored to students' prior knowledge levels: worked examples for low prior knowledge, faded examples for moderate prior knowledge, and problem solving for high prior knowledge. The results indicated that this approach was effective in reducing students' cognitive load. However, the study did not find a significant difference in students' problem-solving abilities. This finding contrasts with other studies reporting that differentiated instruction can improve students' problem-solving skills (Lai et al., 2020; Siburian et al., 2022). This inconsistency suggests the need for further investigation to examine the consistency and generalizability of these findings.

This study aims to conduct an empirical replication of the study by Aini and Retnowati (2023) by focusing on two aspects: (1) testing the generalizability of the findings across different participants and subject domains, and (2) modifying the treatment for the high-ability group by replacing the problem-solving method with the problem-example method. This modification is based on findings that the problem-example method has been proven effective for high-ability (Reisslein et al., 2006). In this method, learners are first allowed to solve problems independently, and subsequently study worked examples after encountering difficulties. This approach is grounded in the assumption that initial failure experiences can enhance learners' motivation and focus when studying examples (Hausmann et al., 2008; Reisslein et al., 2006; Renkl et al., 1998).

LITERATURE REVIEW

Cognitive Load Theory

Cognitive Load Theory (CLT) provides a robust theoretical foundation for understanding how information is processed within the human cognitive architecture. The theory posits that working memory has a strictly limited capacity for processing novel information before it can be encoded and transferred into long-term memory (Sweller, 2024). Variations in the schema structures stored within long-term memory explain why experts solve problems far more efficiently than novices. Experts can automatically recognize problem patterns and solution procedures based on prior experiences, whereas novices must rely on random trial-and-error or means-end analysis, which inherently yields a higher cognitive load. Consequently, instructional strategies that immediately demand independent problem-solving are highly likely to induce cognitive overload in students with limited prior knowledge

Furthermore, CLT emphasizes that the effectiveness of an instructional strategy is relative to the learner's level of expertise (Kalyuga, 2007). This phenomenon is known as the expertise reversal effect, wherein instructional guidance (such as worked examples) that is highly effective for novice learners becomes redundant and ineffective for learners who have already acquired mature cognitive schemas (Kalyuga et al., 2001; Sweller et al., 2011). In the initial stages of learning, worked examples are essential to direct students' attention toward proper solution structures, thereby minimizing extraneous cognitive load and constructing foundational problem-solving schemas (Sweller et al., 1998, 2011; Putri et al., 2025; Santosa et al., 2025).

However, as cognitive schemas develop, continuously presenting fully worked-out solutions introduces informational redundancy, which ceases to facilitate cognitive independence. To bridge the transition from guided instruction to independent problem-solving, the faded-example approach serves as a critical intermediate strategy. This method presents worked examples that progressively omit solution steps, thereby prompting students to actively complete the remaining procedures (Renkl et al., 2004). Through this systematic removal of scaffolding, students shift from observing solutions to independently constructing knowledge. For students with high prior knowledge, a problem-example approach becomes more appropriate. In this approach, students are first confronted with a problem independently and are subsequently provided with a solution example for reflection. This strategy fosters schema activation, solution evaluation, and the optimization of germane cognitive load through internal elaboration and conceptual processing.

Differentiated Instruction

Given that differences in students' prior knowledge require distinct instructional alignments, differentiated instruction (DI) serves as an ideal pedagogical framework. Differentiated instruction emphasizes the proactive adaptation of instructional processes, content, and products to accommodate students' readiness, interests, and learning profiles (Tomlinson et al., 2003). Within the scope of managing student cognitive load, differentiation can be strategically focused on the process dimension. This is achieved by assigning worked-examples to low prior ability students, faded-examples to moderate prior ability students, and problem-examples to high prior ability students.

Conceptually, the theoretical framework of this study indicates that variations in students' prior knowledge directly implicate their cognitive load boundaries and the corresponding effectiveness of instructional designs. By implementing a CLT-based differentiated instructional process, each student receives a learning intervention tailored precisely to their current cognitive capacity. This harmonized approach is conceptualized not only to enhance mathematical problem-solving performance but also to optimize working memory efficiency by minimizing extraneous load and promoting deep conceptual integration.

METHODOLOGY

Design

This study used a Randomized Pretest-Posttest Control Group Design (Fraenkel et al., 2012). In this design, two groups of subjects were used, the experimental group and the control group, and both were measured twice, before the intervention (pretest) and after the intervention (posttest) (Table 1).

Table 1. Randomized Pretest-Posttest Control Group Design

Experimental group	***R***	***O***	***X***	***O***
Control group	*R*	*O*	*C*	*O*

The intervention administered to the experimental group was differentiated instruction based on prior ability and grounded in Cognitive Load Theory (CLT). In contrast, the control group received conventional instruction without differentiated learning.

Participants

This study involved 84 students (aged 14–15 years) in the final grade of a junior high school in Serang, Indonesia. From a total population of 334 students, two classes were randomly selected and assigned to the group receiving CLT-based differentiated instruction (n = 42) and the group receiving non-differentiated instruction (n = 42). The participants took part voluntarily and completed the intervention.

Procedure

The intervention was conducted in the eighth week of the first semester to ensure that the experimental material coincided with its introduction in regular classroom instruction. At the beginning of the study (Time 1), students completed a prior ability test covering prerequisite learning materials, a pretest of mathematical problem-solving ability, and the Rating Scale of Mental Effort (RSME). Subsequently, students completed four worksheets in groups over a period of two to three weeks. In the experimental group, differentiated instruction (DI) was implemented, with group formation based on students' prior ability categories, whereas in the control group, differentiated instruction was not implemented (non-DI), and groups were formed heterogeneously. The teacher allocated approximately 60 minutes for each worksheet. After completing the four worksheets (Time 2), students completed the posttest of mathematical problem-solving ability and the RSME.

Instrumen

To answer the research questions, four instruments were administered to the participants, namely the prior ability test, the mathematical problem-solving ability test, the rating scale of mental effort, and student worksheets.

Prior ability test. The test was developed by the researcher based on prerequisite materials required to learn the topic of geometric transformation, including Cartesian coordinates and functions. The test consisted of 12 essay items and showed reliability (Cronbach's α = 0.1758). The test was used to classify students into high, medium, and low prior ability categories in the differentiated instruction (DI) group.

Mathematical problem-solving ability test. Six items were administered to students, consisting of a combination of well-structured problems (Jonassen, 1997), word problems (Tao et al., 2025), and geometric illustrations. The items were developed based on four problem-solving indicators according to Pólya (2004), understanding the problem, devising a plan, carrying out the plan, and looking back. Before implementation, the instrument was tested for validity through expert judgement and was piloted on 46 students outside the research sample to determine its reliability level. The results showed that the reliability coefficient was 0.6882, indicating that the instrument had a high level of reliability.

Rating scale of mental effort (RSME). This subjective rating scale was used to measure the level of mental load or mental effort invested by students while completing the pretest and posttest of mathematical problem-solving ability. In this study, the RSME instrument was adopted from the scale developed by Paas and Van Merriënboer (1993, 1994) and adapted by Santosa et al. (2018), with a rating range of 1–9 (Figure 1), which has been validated in previous studies and shows good reliability.

1	2	3	4	5	6	7	8	9
Very, very low mental effort	Very low mental effort	Low mental effort	Rather low mental effort	Neither low nor high mental effort	Rather high mental effort	High mental effort	Very high mental effort	Very, very high mental effort

Figure 1. Rating scale of mental effort (adapted from Santosa et al., 2018)

Student worksheets. Worksheets were used as supporting instruments in the intervention process. In the experimental group, the worksheets were developed based on Cognitive Load Theory principles by considering instructional design effects, such as the split-attention effect and redundancy effect (Figure 2), and were arranged in three forms adjusted to students' prior ability: worked-example (low ability), faded-example (medium ability), and problem-example (high ability) (Figure 3). In the control group, the worksheets were presented in the form of problem solving that was not designed based on Cognitive Load Theory principles.

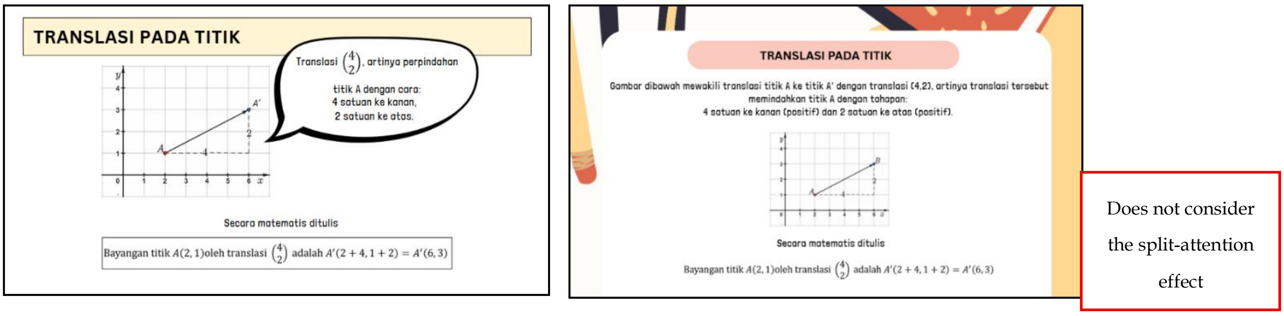


Figure 2. Material presentation design (a) applying Cognitive Load Theory principles and (b) not applying Cognitive Load Theory principles

RESULT AND DISCUSSION

Analysis of Mathematical Problem-Solving Ability

Students' mathematical problem-solving ability (MPSA) in the differentiated instruction (DI) group and the group without differentiated instruction (non-DI) was analyzed using pretest, posttest, and N-Gain data.

Table 2. Mathematical Problem-Solving Ability Based on Learning Groups

	Group	N	Min	Max	Mean	Median	Std. Deviation
Pretest MPSA	DI	42	0	11	5,02	5,00	3,096
	Non-DI	42	0	10	3,90	3,00	2,721
Posttest MPSA	DI	42	4	24	15,76	17,00	6,581
	Non-DI	42	3	22	11,48	12,00	5,447

Table 2 presents descriptive statistics of MPSA data in both groups before (pretest) and after (posttest) the intervention was administered. Based on the descriptive analysis results, the mean pretest score in the DI group was 5.02 ($SD = 3.096$), while in the non-DI group it was 3.90 ($SD = 2.721$). Although descriptively there was a difference in means, the equivalence of initial ability between the DI and non-DI groups was tested using the Mann-Whitney test on pretest MPSA data. The normality test using Shapiro-Wilk showed that the distribution of pretest MPSA data in both groups did not meet the normality assumption (DI group: $W = 0.935$, $p = 0.020$; non-DI group: $W = 0.837$, $p < 0.001$) (Table 3), therefore a nonparametric test was used. The results of the Mann-Whitney test showed that there was no significant difference between the pretest MPSA scores of the DI and non-DI groups ($U = 697.500$; $Z = -1.663$; $p = 0.096 > 0.05$) (Table 4), confirming that both groups had equivalent initial ability conditions before the intervention was implemented.

Table 3. Results of Normality Test of Mathematical Problem-Solving Ability

	Group	Shapiro-Wilk	df	Sig.	H0
Pretest MPSA	DI	0,935	42	0,020	Rejected
	Non-DI	0,837	42	<0,001	Rejected
Posttest MPSA	DI	0,906	42	0,002	Rejected
	Non-DI	0,927	42	0,010	Rejected
N-Gain	DI	0,938	42	0,533	Accepted

MPSA	Non-DI	0,977	42	0,024	Rejected

Note: * Significant at $\alpha = 0.05$; H_0: data are normally distributed

Table 4. Results of Mann-Whitney Test of Mathematical Problem-Solving Ability

	Mann-Whitney U	Wilcoxon W	Z	Sig. (2-tailed)	H0
Pretest MPSA	697,500	1600,500	-1,663	0,096	Accepted
Posttest MPSA	537,000	1440,0001	-3,093	0,002*	Rejected
N-Gain MPSA	530,500	433,500	-3,145	0,002*	Rejected

Note: * Significant at $\alpha = 0.05$; H_0: there is no significant difference between the two groups

Further analysis of posttest MPSA data using the Mann-Whitney test resulted in U = 537.000, Z = -3.093, and Asymp. Sig. (2-tailed) = 0.002 ($p < 0.05$) (Table 4). These results indicate a significant difference in students' mathematical problem-solving ability between the DI and non-DI groups after the intervention. Descriptively, the DI group obtained a higher mean posttest score (MDI = 15.76; SDDI = 6.581) compared to the non-DI group (Mnon-DI = 11.48; SDnon-DI = 5.447).

Subsequently, an analysis of normalized gain (N-Gain) scores of MPSA was conducted. The use of N-Gain was selected because it accounts for differences in initial scores among individuals, allowing a more accurate comparison of improvement between groups. The Mann-Whitney test on N-Gain MPSA resulted in U = 530.500, Z = -3.145, and Asymp. Sig. (2-tailed) = 0.002 ($p < 0.05$), indicating a significant difference between the two groups, with the mean N-Gain of the DI group being 0.584 (SD = 0.322), while the non-DI group was 0.375 (SD = 0.260). These findings indicate that the improvement in mathematical problem-solving ability in the DI group is statistically greater than that in the non-DI group.

Analysis of Cognitive Load

Students' cognitive load data were analyzed to examine the effect of differentiated instruction in managing cognitive load based on the principles of Cognitive Load Theory (CLT). Descriptive statistics of cognitive load data in both groups are presented in Table 5.

Table 5. Descriptive Statistics of Students' Cognitive Load

	Group	N	Min	Max	Mean	Median	Std. Deviation
Pretest CL	DI	42	26	54	47,67	50,50	7,895
	Non-DI	42	39	54	49,43	51,000	3,788
Posttest CL	DI	42	8	53	31,64	32,00	12,197
	Non-DI	42	25	53	41,60	43,00	7,902

Table 5 shows that the mean initial cognitive load (pretest) of both groups was relatively equivalent, with the non-DI group having a mean of 49.43 (SD = 3.788) and the DI group 47.67 (SD = 7.895). The relatively large difference in standard deviation in the DI group indicates more heterogeneous variability in initial cognitive load. After the intervention, a decrease in the mean cognitive load occurred in both groups. However, the reduction in the DI group was more substantial, decreasing by 16.03 to 31.64 (SD = 12.197), whereas the non-DI group only decreased by 7.83 to 41.60 (SD = 7.902). The minimum posttest cognitive load score in the DI group reached 8, which is much lower than the minimum score in the non-DI group (25). This indicates that some students in the DI group achieved a very low cognitive load condition after participating in CLT-based differentiated instruction.

Table 6. Summary of Normality Test Results of Cognitive Load Data

	Kelompok	**Shapiro-Wilk**	**df**	**Sig.**
Pretest CL	DI	0,700	42	<0,001
	Non-DI	0,896	42	0,001
Posttest CL	DI	0,934	42	0,017
	Non-DI	0,954	42	0,088
N-Gain CL	DI	0,515	42	<0,001
	Non-DI	0,278	42	<0,001

Note: * Significant at α = 0.05; H_0: data are normally distributed

The normality test of cognitive load data is presented in Table 6. The results of the Shapiro-Wilk normality test show that the distribution of pretest cognitive load data in both the DI group (W = 0.896; p = 0.001) and the non-DI group (W = 0.896; p = 0.001) did not meet the normality assumption, therefore nonparametric analysis was used. Preliminary analysis of pretest cognitive load data using the Mann-Whitney U test (Table 7) showed no significant difference between the two groups (U = 853.500; Z = −0.256; p = 0.798). These results indicate that the initial cognitive load levels of students in both groups were equivalent, indicating that the differences observed after the intervention can be attributed to the effect of the instructional intervention.

Table 7. Results of Mann-Whitney Test of Cognitive Load

	Mann-Whitney U	**Wilcoxon W**	**Z**	**Sig. (2-tailed)**	**H0**
Pretest CL	853,500	1756,500	-0,256	0,798	Accepted
Posttest CL	423,000	1326,000	-4,109	<0,001*	Rejected
N-Gain CL	660,000	1563,000	-1,986	0,047*	Rejected

Note: * Significant at α = 0.05; H_0: there is no significant difference between the two groups

Furthermore, to examine whether there was a difference in cognitive load levels between the two groups after the intervention, the N-Gain cognitive load data were analyzed. The results of the Mann-Whitney U test on N-Gain cognitive load data showed a significant difference between the DI and non-DI groups (U = 660.000; Z = −1.986; p = 0.047) (Table 7). These results indicate that students who received CLT-based differentiated instruction experienced a statistically greater reduction in cognitive load compared to students who received non-differentiated instruction.

Effect of Differentiated Instruction on Mathematical Problem-Solving Ability

The higher mathematical problem-solving performance in the CLT-based differentiated instruction (DI) group can be explained by the alignment between instructional methods and students' prior ability levels. The use of worked-example for students with low prior ability is consistent with the schema acquisition mechanism in CLT. Students with limited prior knowledge tend to allocate substantial working memory resources to strategy search, which restricts their capacity to process mathematical content (Sweller et al., 2011). This condition is associated with misconceptions and unsystematic solution procedures (Rahmaniyah & Ihsanudin, 2024). The use of worked-examples reduces the need for strategy search by providing structured solution steps to the final answer (Figure 3), allowing cognitive resources to be directed toward understanding the rationale of each step. This process reflects the borrowing and reorganizing principle (Sweller et al., 2011), where learning from existing schemas supports efficient knowledge construction and avoids cognitively demanding random processes.

Students' work on worked-example worksheets (Figure 3) indicates a pattern consistent with schema acquisition. The application of similar solution structures to isomorphic problems reflects schema automation, where repeated exposure to coherent examples reduces cognitive demand in subsequent problem solving. This finding is consistent with Hafidhoh et al. (2026), who reported that worked examples enable students to transfer solution procedures from example problems to similar mathematical tasks.

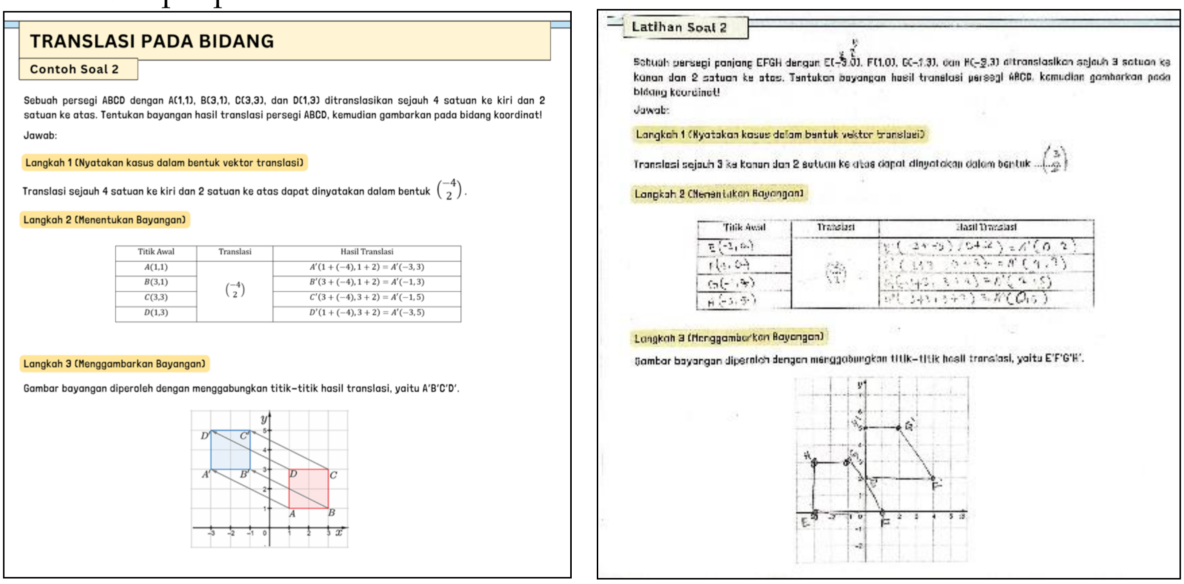


Figure 3. Students' Work on Worked-Example Worksheets

The use of faded examples for students with moderate prior ability supports a gradual transition from example-based learning to independent problem solving. The progressive removal of solution steps (Figure 4) increases cognitive demand in line with schema development (Renkl et al., 2004). This approach addresses the expertise reversal effect, where fully worked examples become less effective and introduce unnecessary cognitive load due to redundant information (Kalyuga et al., 2001). The fading structure allows instruction to adjust to students' competence, increasing opportunities for independent problem solving and supporting the development of more stable schemas (Atkinson et al., 2003).

Students' work on faded-example worksheets (Figure 4) shows the use of solution cues from isomorphic problems as a basis for constructing solution steps independently. The gradual reduction of guidance increases cognitive demand and activates germane cognitive load in a controlled manner.

The use of problem-example for students with high prior ability is associated with productive failure and prior knowledge activation. Requiring students to solve problems before presenting solution examples encourages the use of existing knowledge to construct solutions, although initial attempts may be incomplete (Kapur, 2014). This process supports the formation of problem structure through active processing, so that subsequent exposure to solution examples facilitates the integration of new information into existing schemas (Husmann et al., 2008; Reisslein et al., 2006).

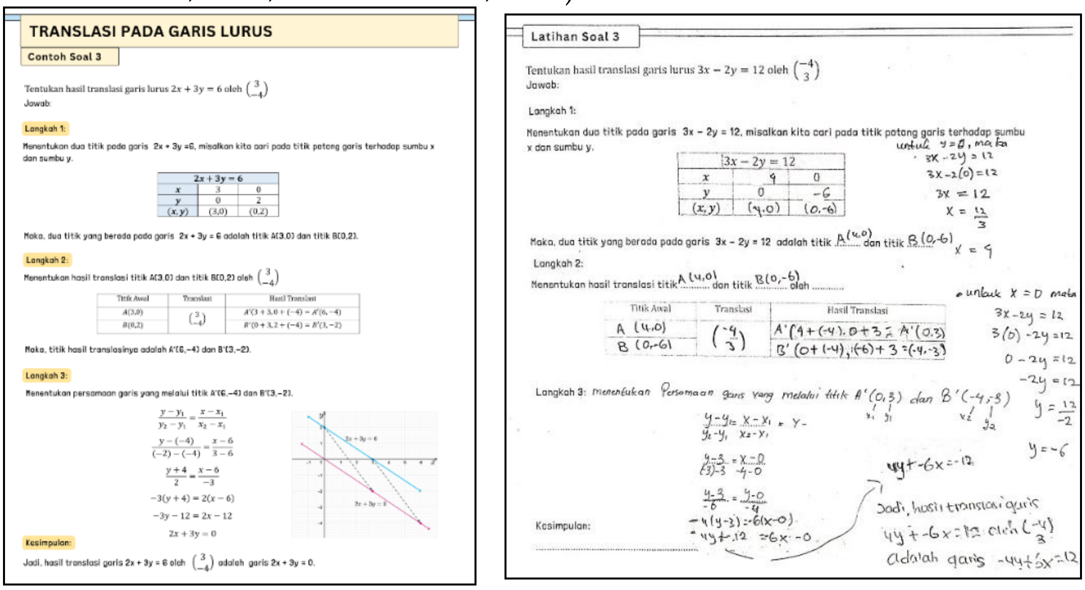


Figure 4. Students' Work on Faded-Example Worksheets

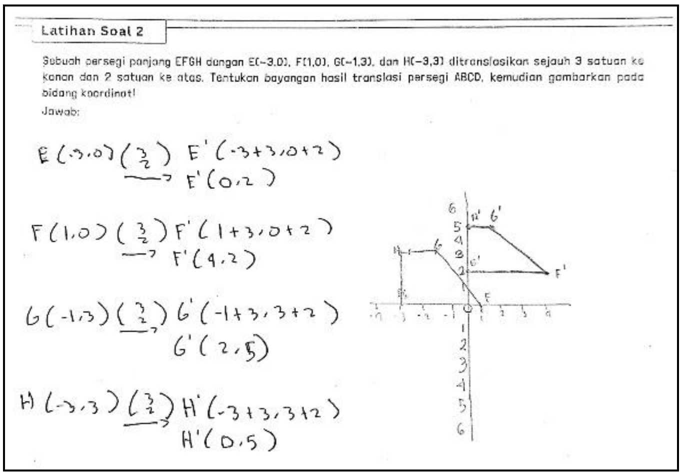
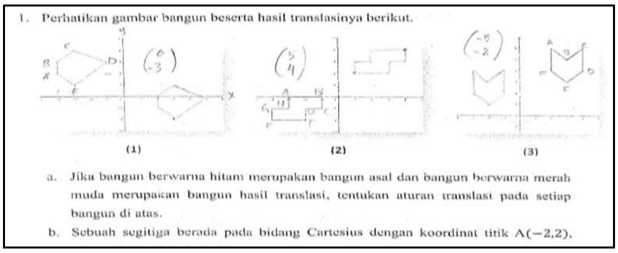


Figure 5. Students' Work on Problem-Example Worksheets

Students using problem-example worksheets (Figure 5) demonstrate greater strategic flexibility compared to those in the worked-example (Figure 3) and faded-example (Figure 4) conditions. Solution procedures are expressed using self-constructed representations. When difficulties occur, students refer to solution examples to evaluate and refine their strategies rather than directly following them.

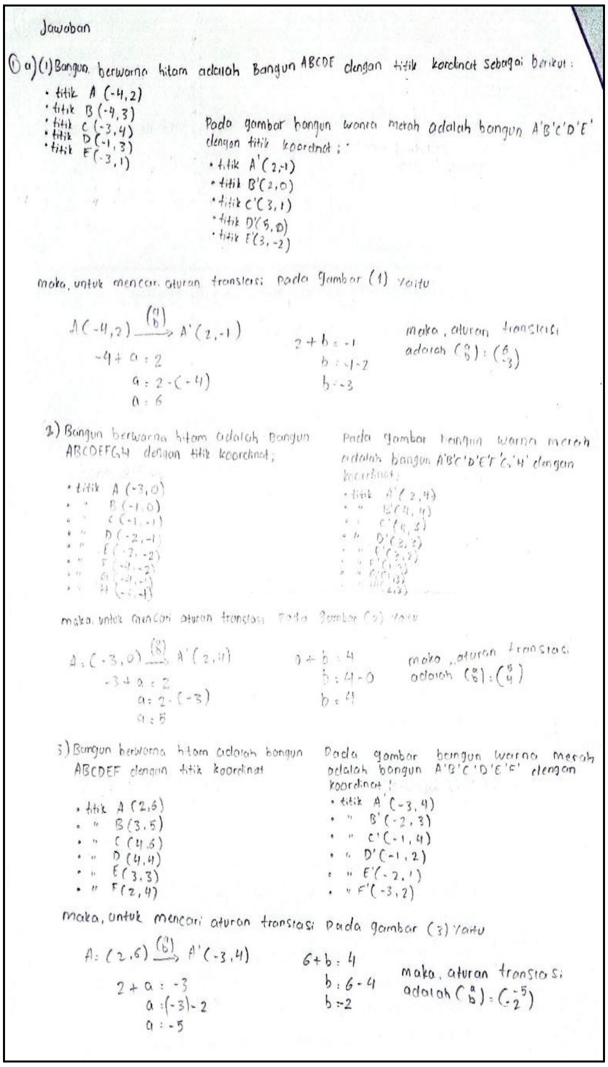


1. Perhatikan gambar bangun beserta hasil translasinya berikut.

(1) (2) (3)

a. Jika bangun berwarna hitam merupakan bangun asal dan bangun berwarna merah muda merupakan bangun hasil translasi, tentukan aturan translasi pada setiap bangun di atas.

b. Sebuah segitiga berada pada bidang Cartesius dengan koordinat titik A(−2,2).



Jawaban

1 a) (1) Bangun berwarna hitam adalah Bangun ABCDE dengan titik koordinat sebagai berikut:

- titik A (-4,2)
- titik B (-4,3)
- titik C (-3,4)
- titik D (-1,3)
- titik E (-3,1)

Pada gambar bangun warna merah adalah bangun A'B'C'D'E' dengan titik koordinat:

- titik A'(2,-1)
- titik B'(2,0)
- titik C'(3,1)
- titik D'(5,0)
- titik E'(3,-2)

maka, untuk mencari aturan translasi pada gambar (1) yaitu

$A(-4,2) \xrightarrow{\binom{a}{b}} A'(2,-1)$

$-4 + a = 2$
$a = 2 - (-4)$
$a = 6$

$2 + b = -1$
$b = -1 - 2$
$b = -3$

maka, aturan translasi adalah $\binom{a}{b} = \binom{6}{-3}$

2) Bangun berwarna hitam adalah Bangun ABCDEFGH dengan titik koordinat;

- titik A (-3,0)
- " B (-1,0)
- " C (-1,-1)
- " D (-2,-1)
- " E (-2,-2)
- " F (-4,-2)
- " G (-4,-1)
- " H (-3,-1)

Pada gambar bangun warna merah adalah bangun A'B'C'D'E'F'G'H' dengan koordinat;

maka, untuk mencari aturan translasi pada gambar (2) yaitu

$A: (-3,0) \xrightarrow{\binom{a}{b}} A'(2,4)$

$-3 + a = 2$
$a = 2 - (-3)$
$a = 5$

$0 + b = 4$
$b = 4 - 0$
$b = 4$

maka, aturan translasi adalah $\binom{a}{b} = \binom{5}{4}$

3) Bangun berwarna hitam adalah bangun ABCDEF dengan titik koordinat

- titik A (2,6)
- " B (3,5)
- " C (4,6)
- " D (4,4)
- " E (3,3)
- " F (2,4)

Pada gambar bangun warna merah adalah bangun A'B'C'D'E'F' dengan koordinat:

- titik A'(-3,4)
- " B'(-2,3)
- " C'(-1,4)
- " D'(-1,2)
- " E'(-2,1)
- " F'(-3,2)

maka, untuk mencari aturan translasi pada gambar (3) yaitu

$A: (2,6) \xrightarrow{\binom{a}{b}} A'(-3,4)$

$2 + a = -3$
$a = (-3) - 2$
$a = -5$

$6 + b = 4$
$b = 6 - 4$
$b = -2$

maka, aturan translasi adalah $\binom{a}{b} = \binom{-5}{-2}$

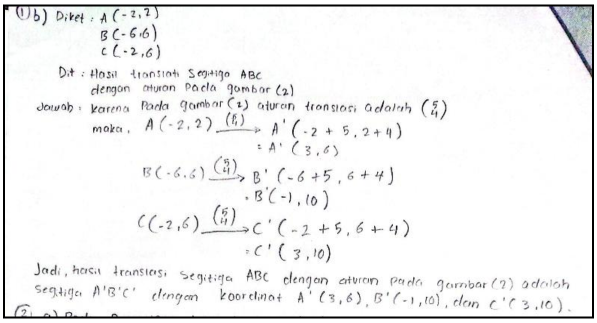


Figure 6. Posttest MPSA Answer of ST-1

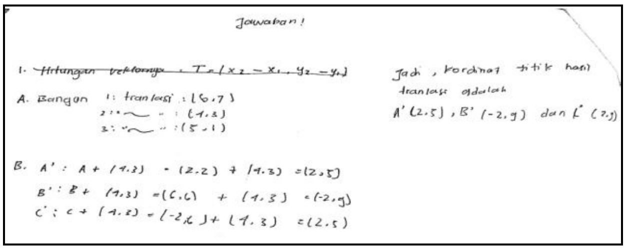


Figure 7. Posttest MPSA Answer of ST-2

Further analysis of representative responses from the DI group (ST-1) and the non-DI group (ST-2) is presented in Figure 6 and Figure 7. In the DI group, ST-1 demonstrates a systematic and complete problem-solving process. The student identifies given and required information, determines the translation vector accurately, and applies the solution strategy consistently, resulting in correct translated coordinates. The solution is concluded appropriately, indicating metacognitive awareness in verifying the result. In the non-DI group, ST-2 shows an unstructured process across Polya's stages. The relationship between the coordinates of the original and translated figures is not identified, and the translation vector is not derived from the given information. The solution is presented without a clear strategy and includes incorrect application of translation rules. This leads to errors in determining the translated coordinates. The process ends without verification, leaving incorrect results uncorrected.

The contrasting patterns reflect the effect of differentiated instructional processes in the DI group. CLT-based differentiated instruction, aligned with prior ability, supports schema development that enables students to carry out Polya's stages more systematically. In contrast, conventional instruction in the non-DI group does not provide sufficient scaffolding, increasing the risk of cognitive overload and limiting the development of coherent solution strategies. This finding is consistent with Kalyuga (2007), who emphasizes the need to adjust instructional methods and levels of guidance based on students' expertise.

Effect of Differentiated Instruction on Students' Cognitive Load

CLT-based differentiated instruction reduces students' cognitive load compared to non-differentiated instruction. These findings are consistent with Aini and Retnowati (2023), who reported that CLT-based differentiated instruction effectively reduced students' cognitive load. The consistency of these findings across different participants and learning contexts strengthens the empirical support for the effectiveness of CLT-based differentiated instruction in managing cognitive load. This finding is also in line with Cognitive Load Theory, which emphasizes that instructional design should consider the limited capacity of working memory, particularly by reducing extraneous cognitive load (Sweller et al., 1998, 2011).

In the low prior ability group, worked-example reduces extraneous cognitive load and directs attention to relevant solution steps, eliminating inefficient search strategies. This allows working memory resources to be used for schema construction in long-term memory, supporting more efficient retention and transfer of knowledge (Nilawati et al., 2025). In the moderate prior ability group, faded-example supports a gradual shift from guided examples to independent processing. This shift increases cognitive demand in a controlled manner and activates germane cognitive load, representing desirable difficulty that supports deeper processing. In the high prior ability group, problem-example reduces the redundancy effect associated with the expertise reversal effect. Presenting fully worked steps to learners with established schemas introduces unnecessary cognitive load because the information overlaps with existing knowledge (Kalyuga et al., 2001). Starting with independent problem solving allows prior knowledge to be used more efficiently before confirmation through solution examples.

CONCLUSIONS AND RECOMMENDATIONS

This study confirms that CLT-based differentiated instruction has a significant effect on students' mathematical problem-solving ability and cognitive load compared to non-differentiated instruction. These findings extend the generalizability of the study by Aini and Retnowati (2023) to different contexts and populations, while also indicating that the modification of the problem-example method for students with high prior ability is an effective alternative within the framework of CLT-based differentiated instruction. Based on these findings, this CLT-based differentiated instruction approach is highly recommended to be considered as an alternative instructional strategy in mathematics education, particularly to enhance students' problem-solving performance while simultaneously managing their cognitive load in heterogeneous classrooms.

FURTHER STUDY

A primary limitation of this research concerns the measurement of cognitive load, which relied solely on the Rating Scale of Mental Effort (RSME), a subjective self-report instrument. Although widely validated, self-report scales are inherently susceptible to student response bias, where participants may over- or underestimate their actual cognitive investment based on social desirability or subjective perceptions. To strengthen future findings, it is highly recommended that subsequent studies employ physiological measurement methods, such as eye-tracking, electroencephalography (EEG), or heart rate variability (HRV), alongside subjective scales. Combining these objective, real-time physiological indicators with self-report measures would provide a more robust, multi-dimensional, and precise triangulation of students' cognitive load during mathematical problem-solving tasks.

In addition, the study was conducted in a single school and focused exclusively on geometric transformation topics. Therefore, caution should be exercised when generalizing the findings to different educational contexts, mathematical domains, and age groups.

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REFERENCES

Aini, R. N., & Retnowati, E. (2023). Apakah pembelajaran diferensiasi mengakibatkan cognitive load tinggi? Jurnal Pengembangan Pembelajaran Matematika, 5(2), 88–103.

Alashwal, H. A., & Barham, A. I. (2025). Sustaining problem-based learning: A mixed-methods exploration of its long-term effects on primary students' mathematical problem solving. Social Sciences & Humanities Open, 12, 101717–101717. https://doi.org/10.1016/j.ssaho.2025.101717

Amalina, I. K., Vidákovich, T., Akhmetova, A., & Brečka, P. (2026). Integrated STEM-based mathematical problem-solving skills of students: The role of prior mathematical knowledge and text comprehension skills. Thinking Skills and Creativity, 61, 102149. https://doi.org/10.1016/j.tsc.2026.102149

Atkinson, R. K., Derry, S. J., Renkl, A., & Wortham, D. (2000). Learning from Examples: Instructional Principles from the Worked Examples Research. Review of Educational Research, 70(2), 181–214.

Atkinson, R. K., Renkl, A., & Merrill, M. M. (2003). Transitioning From Studying Examples to Solving Problems: Effects of Self-Explanation Prompts and Fading Worked-Out Steps. Journal of Educational Psychology, 95(4), 774–783. https://doi.org/10.1037/0022-0663.95.4.774

Booth, J. L., & Davenport, J. L. (2013). The role of problem representation and feature knowledge in algebraic equation-solving. The Journal of Mathematical Behavior, 32(3), 415–423.

Brookhart, S. M., & Nitko, A. J. (2011). Educational assessment of students. Pearson Education.Chen, O., Kalyuga, S., & Sweller, J. (2015). The worked example effect, the generation effect, and element interactivity. Journal of Educational Psychology, 107(3), 689–704.

English, L., & Gainsburg, J. (2016). Problem solving in a 21st-century mathematics curriculum. In Handbook of international research in mathematics education (3rd ed.). Handbook of international research in mathematics education. urn: isbn:978-0-415-83203-8

Fraenkel, J. R., Wallen, N. E., & Hyun, H. (2012). How to design and evaluate education research. (8th ed.). Mcgraw-Hill.Greiff, S., Holt, D. V., & Funke,

J. (2021). Perspectives on Problem Solving in Educational Assessment: Analytical, Interactive, and Collaborative Problem Solving. The Journal of Problem Solving, 5(2). https://doi.org/10.7771/1932-6246.1153

Hafidhoh, R., Santosa, C. A. H. F., & Mutaqin, A. (2026). The Effect of Worked-Examples on Students' Learning Outcomes in Complex Mathematics Materials. *Edumatica: Jurnal Pendidikan Matematika* , *16*(1). https://doi.org/10.22437/edumatica.vl6i1.46293

Hausmann, R., Sande, B., & VanLehn, K. (2008). Are Self-explaining and Coached Problem Solving More Effective When Done by Pairs of Students Than Alone? ArXiv (Cornell University).

Jager, L., Denessen, E., Cillessen, A., & Meijer, P. C. (2022). Capturing instructional differentiation in educational research: investigating opportunities and challenges. Educational Research, 64(2), 224–241.

Jonassen, D. H. (1997). Instructional design models for well-structured and III-structured problem-solving learning outcomes. Educational Technology Research and Development, 45(1), 65–94.

Kalyuga, S. (2007). Expertise Reversal Effect and Its Implications for Learner-Tailored Instruction. Educational Psychology Review, 19(4), 509–539. https://doi.org/10.1007/s10648-007-9054-3

Kalyuga, S., Chandler, P., Tuovinen, J., & Sweller, J. (2001). When problem solving is superior to studying worked examples. Journal of Educational Psychology, 93(3), 579–588. https://doi.org/10.1037/0022-0663.93.3.579

Kapur, M. (2014). Productive Failure in Learning Math. Cognitive Science, 38(5), 1008–1022. https://doi.org/10.1111/cogs.12107

Lai, C.-P., Zhang, W., & Chang, Y.-L. (2020). Differentiated instruction enhances sixth-grade students' mathematics self-efficacy, learning motives, and problem-solving skills. Social Behavior and Personality: An International Journal, 48(6). https://doi.org/10.2224/sbp.9094

Lester, F. K. (2013). Thoughts About Research on Mathematical Problem-Solving Instruction. The Mathematics Enthusiast, 10(1-2), 245–278. https://doi.org/10.54870/1551-3440.1267

López, M. V., García, A., & Rodríguez, L. (2007). Sustainable Development and Corporate Performance: A Study Based on the Dow Jones Sustainability Index. Journal of Business Ethics, 75(3), 285–300.

Nakakoji, Y., & Wilson, R. (2020). Interdisciplinary Learning in Mathematics and Science: Transfer of Learning for 21st Century Problem Solving at University. Journal of Intelligence, 8(3), 32.

NCTM. (2000). Principles and Standards for School Mathematics. National Council of Teachers of Mathematics.

NCTM. (2014). Principles to actions: Ensuring mathematical success for all. National Council of Teachers of Mathematics.

Nilawati, N., Santosa, C. A. H. F., Fakhrudin, & Effendi, A. (2025). Effect of Example-Based Learning Model on Micro Level Cognitive Load and Knowledge Transfer. *RANGE: Jurnal Pendidikan Matematika*, *7*(1), 71–86. https://doi.org/10.32938/jpm.v7i1.9517

Paas, F. G. W. C., & Van Merriënboer, J. J. G. (1993). The Efficiency of

Instructional Conditions: An Approach to Combine Mental Effort and Performance Measures. *Human Factors: The Journal of the Human Factors and Ergonomics Society*, *35*(4), 737–743. https://doi.org/10.1177/001872089303500412

Paas, F. G. W. C., van Merriënboer, J. J. G., & Adam, J. J. (1994). Measurement of Cognitive Load in Instructional Research. Perceptual and Motor Skills, 79(1), 419–430. https://doi.org/10.2466/pms.1994.79.1.419

Pólya, G. (2004). How to solve it: A new aspect of mathematical method. Princeton University Press.

Putri, Z. A. H., Santosa, C. A. H. F., & Novaliyosi. (2025). The Effect of Video Modeling Example, Worked-Example, and Practice Problem on Mathematics Learning . *Edumatica: Jurnal Pendidikan Matematika*, *15*(3). https://doi.org/10.22437/edumatica.v15i3.46296

Rahmaniyah, Z., & Ihsanudin, I. (2024). Analysis of Junior High School Students' Misconceptions Based on Constructivism Theory in Solving Numeracy Literacy Problems. Alpha Math: Journal of Mathematics Education, 10(2), 284. https://doi.org/10.30595/alphamath.v10i2.21791

Reisslein, J., Atkinson, R. K., Seeling, P., & Reisslein, M. (2006). Encountering the expertise reversal effect with a computer-based environment on electrical circuit analysis. Learning and Instruction, 16(2), 92–103.

Renkl, Alexander; Atkinson, Robert K.; Maier, Uwe H. (2000). From Studying Examples to Solving Problems: Fading Worked-Out Solution Steps Helps Learning. 22.

Renkl, A., Atkinson, R. K., & Große, C. S. (2004). How Fading Worked Solution Steps Works – A Cognitive Load Perspective. Instructional Science, 32(1/2), 59–82. https://doi.org/10.1023/b:truc.0000021815.74806.f6

Renkl, A., Atkinson, R. K., Maier, U. H., & Staley, R. (2002). From Example Study to Problem Solving: Smooth Transitions Help Learning. The Journal of Experimental Education, 70(4), 293–315.

Renkl, A., Stark, R., Gruber, H., & Mandl, H. (1998). Learning from Worked-Out Examples: The Effects of Example Variability and Elicited Self-Explanations. Contemporary Educational Psychology, 23(1), 90–108.

Rosinta Siburian, Sinta Dameria Simanjuntak, & Frida Marta Simorangkir. (2022). Effectiveness of online differentiated instruction in terms of students' mathematical problem-solving ability. Jurnal Riset Pendidikan Matematika/Jurnal Riset Pendidikan Matematika, 9(1).

Santosa, C. A. H. F. (2019). Fostering Germane Load Through Self-Explanation Prompting In Calculus Instruction. *IJOLAE: Indonesian Journal on Learning and Advanced Education*, *1*(1). https://doi.org/10.23917/ijolae.v1i1.7421

Santosa, C. A. H. F., & Mehmet Filiz. (2025). Investigating the limit of peer collaboration: Insight from worked-example in multivariable calculus. *Infinity Journal*, *14*(2), 461–482. https://doi.org/10.22460/infinity.v14i2.p461-482

Santosa, C. A. H. F., Suryadi, D., Prabawanto, S., & Syamsuri, S. (2018). The role of worked-example in enhancing students' self-explanation and cognitive efficiency in calculus instruction. Jurnal Riset Pendidikan Matematika, 5(2),

168–180. https://doi.org/10.21831/jrpm.v0i0.19602
Smale-Jacobse, A. E., Meijer, A., Helms-Lorenz, M., & Maulana, R. (2019). Differentiated instruction in secondary education: A systematic review of research evidence. Frontiers in Psychology, 10(2366).
Suprayogi, M. N., Valcke, M., & Godwin, R. (2017). Teachers and Their Implementation of Differentiated Instruction in the Classroom. Teaching and Teacher Education, 67, 291–301.
Süß, H.-M., & Kretzschmar, A. (2018). Impact of Cognitive Abilities and Prior Knowledge on Complex Problem-Solving Performance – Empirical Results and a Plea for Ecologically Valid Microworlds. Frontiers in Psychology, 9.
Sweller, J. (2024). Cognitive Load Theory and Individual Differences. Learning and Individual Differences, 110(1), 102423–102423.
Sweller, J., Ayres, P., & Kalyuga, S. (2011a). Cognitive Load Theory. Springer Science+Business Media, LLC, Cop.Sweller, J., Ayres, P., & Kalyuga, S. (2011b). The Worked Example and Problem Completion Effects. Cognitive Load Theory, 99–109. https://doi.org/10.1007/978-1-4419-8126-4_8
Sweller, J., van Merrienboer, J. J. G., & Paas, F. G. W. C. (1998). Cognitive Architecture and Instructional Design. Educational Psychology Review, 10(3), 251–296. https://doi.org/10.1023/a:1022193728205
Tao, X., Zhang, Y., Xie, Z., Zhao, Z., Zhou, G., & Lu, Y. (2025). Unifying the syntax and semantics for math word problem solving. Neurocomputing, 636, 130042. https://doi.org/10.1016/j.neucom.2025.130042
Ter Vrugte, J., de Jong, T., Vandercruysse, S., Wouters, P., van Oostendorp, H., & Elen, J. (2017). Computer game-based mathematics education: Embedded faded worked examples facilitate knowledge acquisition. Learning and Instruction, 50, 44–53.
Tomlinson, C. A., Brighton, C., Hertberg, H., Callahan, C. M., Moon, T. R., Brimijoin, K., Conover, L. A., & Reynolds, T. (2003). Differentiating Instruction in Response to Student Readiness, Interest, and Learning Profile in Academically Diverse Classrooms: A Review of Literature. Journal for the Education of the Gifted, 27(2-3), 119–145. https://doi.org/10.1177/016235320302700203
van Gog, T., Kester, L., & Paas, F. (2011). Effects of worked examples, example-problem, and problem-example pairs on novices' learning. Contemporary Educational Psychology, 36(3), 212–218.
Whitley, J., Gooderham, S., Duquette, C., Orders, S., & Cousins, J. B. (2019). Implementing differentiated instruction: a mixed-methods exploration of teacher beliefs and practices. Teachers and Teaching, 25(8), 1–19.